BRIDGE STATISTICS

HIGH CARD POINT DISTRIBUTIONS

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ABSTRACT

This document presents tables that show the distribution of high card points in bridge hands. These tables were generated by a computer program that simulates shuffling and dealing a deck of cards into four hands.

INTRODUCTION

The total number of different bridge hands is the total number of combinations of 13 cards that can be made from a deck of 52 cards. This total can be calculated using the formula for combinations which is:

$$\frac{52!}{13!\times 39!} = \frac{52\times 51\times 50\times \ldots \times 2}{13\times 12\times 11\times \ldots \times 2 \times 39\times 38\times 37\times \ldots \times 2} = \frac{52\times 51\times 50\times \ldots \times 40}{13\times 12\times 11\times \ldots \times 2}$$

The result is **635,013,559,600** different bridge hands that you might see in your lifetime.

Some of the questions that we might ask about the distribution of these hands can be easily answered using the mathematics for combinations and probabilities. But the answers to some of our questions might require mathematics that are complex and difficult to understand.

Another way to answer these questions is to write a computer program that generates all 635+ billion possible hands so that precise counts can be obtained for hands with a specific point count or card distribution. However, such a program would take a very long time to run.

If we are willing to settle for approximate answers to our questions, the computer can be programmed to simulate shuffling a deck of 52 cards and dealing the shuffled cards into four hands. These hands can then be analyzed to answer our questions about the distribution of the hands. This also allows the analysis of a bridge table to answer questions about the distribution of hands in a partnership. If we then ask the computer to do this a large but manageable number of times, we can obtain counts and percentages that are a close approximation to the precise mathematical results. All of the statistics presented in the following sections are based on this technique.

The computer simulation strives to shuffle the deck in a completely random fashion. But after a real hand of bridge is played, the cards are grouped by trick which means that when the deck is reassembled there are clumps of cards from the same suit. Since in practice there are usually only three or four shuffles before the next deal, the cards will not be completely randomized. So your real world experience will often differ from those that are expected using mathematics, logic, or random simulations.

SINGLE HAND HIGH CARD POINT DISTRIBUTION

Table 1 shows the distribution of high card points from ten million bridge hands that were simulated by the computer.

Table 1: POINT DISTRIBUTION FROM TEN MILLION HANDS

HIGH CARD POINTS	HAND COUNT	PERCENT OF TOTAL HANDS	CUMULATIVE PERCENT		ONCE EVERY *N* HANDS	HANDS OUT OF 24
			Ascending	Descending		
0	36,239	0.3624	0.3624	100.0000	275.9	
1	78,819	0.7882	1.1506	99.6376	126.9	
2	135,317	1.3532	2.5038	98.8494	73.9	
3	246,334	2.4633	4.9671	97.4962	40.6	1
4	384,608	3.8461	8.8132	95.0329	26.0	1
5	518,975	5.1898	14.0029	91.1868	19.3	1
6	656,173	6.5617	20.5647	85.9971	15.2	1
7	802,145	8.0215	28.5861	79.4353	12.5	2
8	888,540	8.8854	37.4715	71.4139	11.3	2
9	935,733	9.3573	46.8288	62.5285	10.7	3
10	941,059	9.4106	56.2394	53.1712	10.6	2
11	893,717	8.9372	65.1766	43.7606	11.2	2
12	801,934	8.0193	73.1959	34.8234	12.5	2
13	691,614	6.9161	80.1121	26.8041	14.5	2
14	570,548	5.7055	85.8176	19.8879	17.5	1
15	442,763	4.4276	90.2452	14.1824	22.6	1
16	331,699	3.3170	93.5622	9.7548	30.1	1
17	235,993	2.3599	95.9221	6.4378	42.4	1
18	160,465	1.6047	97.5268	4.0779	62.3	
19	103,530	1.0353	98.5621	2.4732	96.6	
20	63,987	0.6399	99.2019	1.4379	156.3	
21	37,396	0.3740	99.5759	0.7981	267.4	
22	21,303	0.2130	99.7889	0.4241	469.4	
23	11,013	0.1101	99.8990	0.2111	908.0	
24 - 26	9,360	0.0936	99.9926	0.1010	1,068.4	
27 - 29	704	0.0070	99.9997	0.0074	14,204.5	
30 - 37	32	0.0003	100.0000	0.0003	312,500.0	
TOTAL	10,000,000	100.0000				23

Note that the simulated percentage of hands with zero points is very close to the actual percentage which is 0.3639% and the percentage of hands with nine points is close to the actual percentage of 9.3562%. The cumulative percent for zero to five points is close to the actual cumulative percent which is 14.0025%.

Table 2 shows the point distribution in three point intervals and *Table 3* shows the distribution in six point intervals for the same hands as *Table 1*.

Table 2: POINT DISTRIBUTION FROM TEN MILLION HANDS

HIGH CARD POINTS	HAND COUNT	PERCENT OF TOTAL HANDS	CUMULATIVE PERCENT		ONCE EVERY *N* HANDS	HANDS OUT OF 24
			Ascending	Descending		
0 - 2	250,375	2.5038	2.5038	100.0000	39.9	
3 - 5	1,149,917	11.4992	14.0029	97.4963	8.7	3
6 - 8	2,346,858	23.4686	37.4715	85.9971	4.3	5
9 - 11	2,770,509	27.7051	65.1766	62.5285	3.6	7
12 - 14	2,064,096	20.6410	85.8176	34.8234	4.8	5
15 - 17	1,010,455	10.1046	95.9221	14.1825	9.9	3
18 - 20	327,982	3.2798	99.2019	4.0779	30.5	
21 - 23	69,712	0.6971	99.8990	0.7981	143.4	
24 - 26	9,360	0.0936	99.9926	0.1010	1,068.4	
27 - 29	704	0.0070	99.9997	0.0074	14,204.5	
30 - 37	32	0.0003	100.0000	0.0003	312,500.0	
TOTAL	10,000,000	100.0000				23

Table 3: POINT DISTRIBUTION FROM TEN MILLION HANDS

HIGH CARD POINTS	HAND COUNT	PERCENT OF TOTAL HANDS	CUMULATIVE PERCENT		ONCE EVERY *N* HANDS	HANDS OUT OF 24
			Ascending	Descending		
0 - 5	1,400,292	14.0029	14.0029	100.0000	7.1	3
6 - 11	5,117,367	51.1737	65.1766	85.9971	2.0	12
12 - 17	3,074,551	30.7455	95.9221	34.8234	3.3	8
18 - 23	397,694	3.9769	99.8990	4.0779	25.1	
24 - 29	10,064	0.1006	99.9997	0.1010	993.6	
30 - 37	32	0.0003	100.0000	0.0003	312,500.0	
TOTAL	10,000,000	100.0000				23

When the simulation is repeated, the counts are slightly different because of the random shuffling of the simulated card deck. But, the percentages are about the same. To demonstrate this fact, the following tables show the results of repeating the simulation.

Table 4: POINT DISTRIBUTION FROM TEN MILLION HANDS

HIGH CARD POINTS	HAND COUNT	PERCENT OF TOTAL HANDS	CUMULATIVE PERCENT		ONCE EVERY *N* HANDS	HANDS OUT OF 24
			Ascending	Descending		
0 - 5	1,400,547	14.0055	14.0055	100.0000	7.1	3
6 - 11	5,117,498	51.1750	65.1805	85.9945	2.0	12
12 - 17	3,072,981	30.7298	95.9103	34.8195	3.3	8
18 - 23	398,887	3.9889	99.8991	4.0897	25.1	
24 - 37	10,087	0.1009	100.0000	0.1009	991.4	

Table 5: POINT DISTRIBUTION FROM TEN MILLION HANDS

HIGH CARD POINTS	HAND COUNT	PERCENT OF TOTAL HANDS	CUMULATIVE PERCENT		ONCE EVERY *N* HANDS	HANDS OUT OF 24
			Ascending	Descending		
0 - 5	1,401,055	14.0105	14.0105	100.0000	7.1	3
6 - 11	5,115,883	51.1588	65.1694	85.9895	2.0	12
12 - 17	3,074,106	30.7411	95.9104	34.8306	3.3	8
18 - 23	398,797	3.9880	99.8984	4.0896	25.1	
24 - 37	10,159	0.1016	100.0000	0.1016	984.3	

Some of the conclusions that can be drawn from these tables are:

- About 65% of hands will have less than 12 HCPs while about 35% of the hands will qualify for an opening bid with 12 or more HCPs. *(See Table 1 or Table 2)*.
- About 10% of hands will have 15-17 HCPs but only 4% of hands will have 18 or more HCPs. *(See Table 2)*
- About one out of every 25 hands will have 18-23 HCPs but only one out of every 143 hands will contain 21-23 HCPs. *(See Table 3 and Table 2)*.
- In a typical session of 24 hands, three hands are likely to contain less than 6 HCPs and three hands are likely to contain 15 or more HCPs. About half of the hands will have 6-11 HCPs. *(See Table 2 and Table 3)*
- Even if you play bridge every day of the week, you will probably never see a hand with 30 or more HCPs.

A REASON TO USE THE WEAK TWO CONVENTION

As *Table 6* indicates, about 56.4% of your hands will have 5-11 HCP's but only 0.8% of your hands will have 21 or more HCP's. This means that if you play 24 hands of bridge you will likely see 13 hands with 5-11 HCP's and no hands with 21 or more HCPs. If you play the ***Weak Two*** convention you might be able to open one of the weaker hands **and** any strong hand that might occur. If you only play ***Strong Twos*** you will not be able to open most of the weaker hands and probably will not get a chance to use the strong two opening bid.

Table 6: POINT DISTRIBUTION FROM TEN MILLION HANDS

High Card Points	0 - 11		12 - 37	
	0 - 4	**5 - 11**	12 - 20	21 - 37
Percent of Deals	8.8%	**56.4%**	34.0%	0.8%
	65.2%		34.8%	

As *Table 7* shows, about 4.3% of your hands will qualify for a weak two opener with 5-11 HCP's, six diamonds or six hearts or six spades, at least two of the top four honors, and no side four card major. If you play 24 bridge hands each week, one of those will likely qualify for a weak two opener while none of them are likely to qualify for a strong two opener.

Table 7: HANDS THAT QUALIFY FOR A WEAK TWO OPENING BID

High Card Points	5 - 11	
	5 - 8	**9 - 11**
Percent of Hands	**1.8%**	**2.5%**
	4.3%	

So the bottom line is the number of hands that qualify for a weak two opener is five times the number of hands that qualify for a strong two opener. Since you are not always in the first seat, you might only get a chance to open about half of the weak hands. But since you can still open all of the rare strong two hands using the ***Strong Two Club*** convention, you give up very little when playing the ***Weak Two*** convention while you increase the number of hands you can open and at the same time make it more difficult for the opponents to find their contract.

PARTNERSHIP HIGH CARD POINT DISTRIBUTION

The distribution of points in one hand is interesting. But since you always play with a partner, the distribution of points in the combined hands of the partnership is much more interesting. *Table 8* shows the partnership distribution from five million deals which creates ten million partnerships.

Table 8: PARTNERSHIP POINT DISTRIBUTION FROM 5,000,000 DEALS

HIGH CARD POINTS	HAND COUNT	PERCENT OF TOTAL HANDS	CUMULATIVE PERCENT		ONCE EVERY *N* HANDS	HANDS OUT OF 24
			Ascending	Descending		
0 - 2	276	0.0028	0.0028	100.0000	36,231.9	
3 - 5	6,842	0.0684	0.0712	99.9972	1,461.6	
6 - 8	61,738	0.6174	0.6886	99.9288	162.0	
9 - 11	300,796	3.0080	3.6965	99.3114	33.2	
12 - 14	895,037	8.9504	12.6469	96.3035	11.2	3
15 - 17	1,763,169	17.6317	30.2786	87.3531	5.7	4
18 - 20	2,384,419	23.8442	54.1228	69.7214	4.2	5
21 - 23	2,244,020	22.4402	76.5630	45.8772	4.5	6
24 - 26	1,466,702	14.6670	91.2300	23.4370	6.8	3
27 - 29	653,528	6.5353	97.7653	8.7700	15.3	2
30 - 32	188,601	1.8860	99.6513	2.2347	53.0	
33 - 35	32,081	0.3208	99.9721	0.3487	311.7	
36 - 38	2,727	0.0273	99.9994	0.0279	3,667.0	
39 - 40	64	0.0006	100.0000	0.0006	156,250.0	
TOTAL	10,000,000	100.0000				23

We can draw the following conclusions from this table:

- A partnership will most likely have 18-23 high cards points. This occurs in about 46% of the deals.
- Assuming it takes at least 24 HCP's for a game, a partnership might be able to make a game in about 23% of the deals. This is about 5 hands out of the 24.
- Assuming it takes at least 33 HCP's for a slam, a partnership will have a slam once in every 287 deals.

PARTNER'S POINT DISTRIBUTION

Because the distribution for the points in your partner's hand will depend on the contents of your hand, it is not possible to come up with a two dimensional table for partner's point distribution. However, a distribution table for partner's hand can be constructed if we specify the characteristics of your hand.

For example, lets suppose that you open one no trump with 15-18 high card points, no voids, no singletons, at most one doubleton, stoppers in at least three suits, and at most one honor if there is a five card major. The following tables give partner's point distribution from the 462,516 hands that could be opened one no trump from a simulation of ten million deals.

Table 9: RESPONDER'S POINT DISTRIBUTION GIVEN A 1-NT OPENING BID

HIGH CARD POINTS	HAND COUNT	PERCENT OF TOTAL HANDS	CUMULATIVE PERCENT	
			Ascending	Descending
0 - 2	24,621	5.3233	5.3233	100.0000
3 - 5	93,019	20.1115	25.4348	94.6767
6 - 8	147,594	31.9111	57.3459	74.5652
9 - 11	122,655	26.5191	83.8650	42.6541
12 - 14	57,587	12.4508	96.3158	16.1350
15 - 17	14,903	3.2222	99.5380	3.6842
18 - 25	2,135	0.4616	99.9996	0.4620
TOTAL	462,514	99.9996		

Table 10: RESPONDER'S POINT DISTRIBUTION GIVEN A 1-NT OPENING BID

HIGH CARD POINTS	HAND COUNT	PERCENT OF TOTAL HANDS	CUMULATIVE PERCENT	
			Ascending	Descending
0 - 7	214,720	46.4243	46.4243	100.0000
8 - 9	97,514	21.0834	67.5077	53.5757
10 - 15	141,076	30.5019	98.0096	32.4923
16 - 25	9,206	1.9904	100.0000	1.9904
TOTAL	462,516	100.0000		

Some conclusions for a responder to one no trump:

- If you were taught that you need at least 8 HCPs to respond to one no trump and you do not play the Jacoby Transfer Convention, you will pass about 46% of times.
- You will have a 44% chance of having enough HCP's to keep the bidding open until a game contract is reached.
- You will be able to direct the partnership to a slam about 2% of the times that your partner opens one no trump.

A REASON TO USE JACOBY TRANSFERS

Table 11 shows the percentage of hands that can be opened one no trump with various high card point ranges.

Table 11: HANDS THAT CAN BE OPENED ONE NO TRUMP

High Card Points	15 - 17	16 - 18	**15 - 18**
Percent of Deals	3.9%	2.9%	**4.6%**

Table 12 shows responders point count after partner opened one no trump with a 15-18 HCP range. The percentages were obtained from *Table 10*. *Table 13* shows the percentage of responders hands that contain a five card or longer major.

Table 12: RESPONDERS POINT COUNT AFTER 1-NT

High Card Points	**0 - 7**	8 - 9	10 - 15	16 - 25
Percent of Hands	**46.4%**	21.1%	30.5%	2.0%

Table 13: RESPONDERS HANDS WITH A FIVE CARD OR LONGER MAJOR

High Card Points	**0 - 7**	8 - 9	10 - 15	16 - 25
Percent of Hands	**15.4%**	5.9%	8.5%	0.5%
	30.3%			

So the bottom line is that if ***Jacoby Transfers*** are not played, responder would usually pass about 46% of the hands that partner opened one no trump. But responder could initiate a ***Jacoby Transfer*** with about 15% of those weak hands or about 30% of all hands.